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ABOUT SOME GEOMETRIC PROBLEMS CONCERNING OPTIMISATION THE OBJECT SHAPE ON THE DESIGN OF A RECTANGULAR POLYGON IN THE ASPECT OF RESOLVING THE ROOF

Scientists, working in the field of monograph [1], are in search of the building shape (symmetrical at least once) polygon with 2*n* vertexes. By modification and defining precisely the area formula and the closing conditions of the polygon, the task for any polygon which is the design of a building, shall be formulated.

Assuming the sequence of the length sides of the polygon $l_1,l_2,\ldots,l_n$ and the sequence $A_1,A_2,\ldots,A_n$ azimuths the surface area of a polygon is expressed with the use of the formula: $P_{pd} = \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} l_i l_j \sin(A_i - A_j)$, with the closing conditions $\sum_{i=1}^{n} l_i \sin A_i = 0$.

The task shall be formulated: given rectangular polygon which is said to be:

1: inscribed in a rectangular with the sides *a* and *b*,

2: the surface area of the polygon is constant and equals *c*,

3: the length sides $l_1,l_2,\ldots,l_n$ satisfy:

3a (constraints): $\underline{l}_i \le l_i \le \bar{l}_i$, for $i=1,2,\ldots,n$,

3b (compatibility conditions): $\sum l_{i_j} = a$, $\sum l_{i_j} = b$.

4: complied with the conditions, for instance $l_{i_j} = l_{i_k}$ for some i_j, i_k.

The argument system is searched l_i for $i=1,2,\ldots,n$, for which objective linear function (*) $P_{sz}(l_1,l_2,\ldots,l_n) = \sum_{i=1}^{n} c_i l_i$ reaches the minimum (P_{sz} – may be interpreted as a surface area of the external buildings sides but after multiplying by the height and taking into account costs c_i as the individual costs of the side or the heat lost per the surface unit).

With the use of the established sides height, the function value (*) is proportional to the total surface of the external building sides which in connection with the constant size of the area *c* and on the assumption that function (*) reaches the minimum gives the best building value. Conditions (3a) determine the acceptable (minimum and maximum) building span and dormer. Conditions 4, with the use of appropriately chosen indexes, determine the building symmetry or its parts. The building symmetry may be "corrected" by widening the roof beyond the outline building sides by creating arcades (Fig. 1a'). The important function that determines the level of the roof complex (rafter framing) is more difficult to formulate as far as our considerations are focused. Figures 1a, 1a', 2a'' are great illustration of the rafter framing complexity with the change of projection shape of the roof for a rectangular dodecagon. The description of the function shall belong to the basic issue as far as the development of problem is concerned.

There is a tendency of leaving the ideal geometric constant building solid in buildings owning the base of rectangular polygon [2]. The introduction of rectangular concave angles (270^o) impoverishes the surface area construction with relation to the material capabilities (Fig. 1). The building lot shape, the building regulations, the span limitation etc, may force the introduction of rectangular concave angles. In such a case, the optimization task formulated above has an infinite number of solutions. If the dimensions of the polygon sides are changed by: Δl_i length l_i polygon side being an arm of an convex angle and reduction by Δl_j length l_j length polygon side being the arm of a concave angle and by providing that the appropriate equation occurs $l_{i+1}\Delta l_i = l_{j-1}\Delta l_j$, the surface area of polygon shall not change.

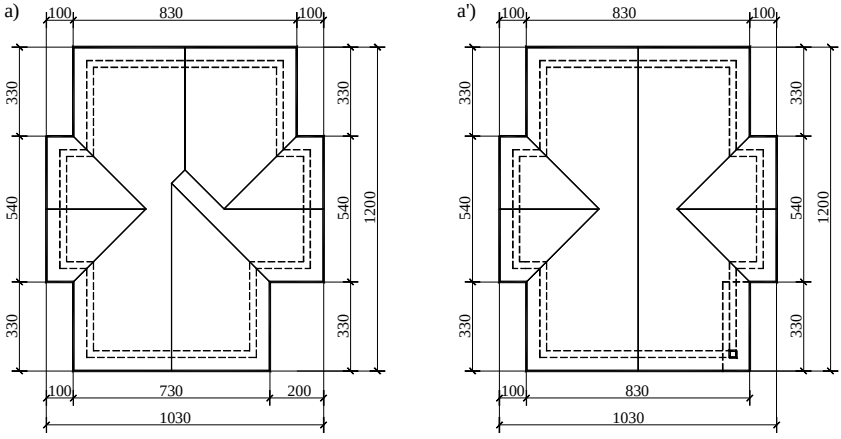

Figure 1: The projection form a building with the roof line solution: a) without taking into account the arcade; a') with taking into account the arcade (the pole supporting the roof in the right-bottom corner)

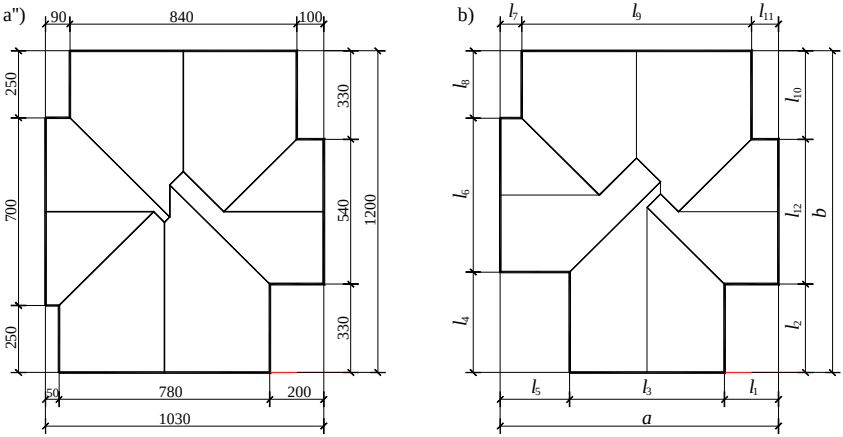

Figure 2: Another type of the roof line solution with: a'') greater complexity; b) designation of the task variable

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[2] Koźniewski E.: *Geometria dachów*. *Teoria i rozwiązanie*. Wydawnictwo Politechniki Białostockiej. Białystok 2007.